Using complete sentences and proper mathematical notation, state the Mean Value Theorem. **SCORE: _____ / 3 PTS**

IF f IS CONTINUOUS ON $[a,b]$ AND f IS DIFFERENTIABLE ON (a,b)
THEN $f'(c) = \frac{f(b)-f(a)}{b-a}$ FOR SOME $c \in (a,b)$

GRADED BY ME

Using complete sentences and proper mathematical notation, state the **formal definition** of "global maximum". **SCORE: _____ / 2 PTS**

f HAS A GLOBAL MAXIMUM ON I AT $c \in I$ IF $f(c) \geq f(x)$ FOR ALL $x \in I$

GRADED BY ME

Consider the following three cases: GRADED BY ME **SCORE: _____ / 8 PTS**

Case 1: $h(x) = x + \frac{1}{x}$ on the interval $[-4, -1]$

Case 2: $f(x) = \cos\frac{1}{x}$ on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$ ← NOT DEFINED/CONTINUOUS @ $x=0$

Case 3: $g(x) = x^{\frac{2}{5}}$ on the interval $[-32, 32]$ ← $g'(x) = \frac{2}{5}x^{-\frac{3}{5}}$ DNE @ $x=0$

[a] In which cases does the Extreme Value Theorem apply ? If the Extreme Value Theorem does **not** apply in any case, write "N/A". For the cases in which the Extreme Value Theorem applies, list all the conditions of the Extreme Value Theorem which are satisfied.

CASES 1,3 : FUNCTIONS ARE CONTINUOUS ON CLOSED + BOUNDED INTERVALS

[b] In which cases does Rolle's Theorem apply ? If Rolle's Theorem does **not** apply in any case, write "N/A". For the cases in which Rolle's Theorem applies, list all the conditions of Rolle's Theorem which are satisfied.

N/A

[c] In which case does the Mean Value Theorem apply ? List all the conditions of the Mean Value Theorem which are satisfied in that case, and find the value of c guaranteed by the theorem.

CASE 1: h IS CONT ON $[-4,-1]$ AND DIFF ON $(-4,-1)$

$$h'(c) = 1 - \frac{1}{c^2} = \frac{(-1+-1)-(-4+-\frac{1}{4})}{-1--4} = \frac{-2+\frac{17}{4}}{3} = \frac{3}{4}$$

$$\frac{1}{4} = \frac{1}{c^2}$$

$c = \pm 2$ $\quad$ $c = -2 \in (-4,-1)$

Find the absolute extrema of $f(x) = x^{\frac{2}{3}}(x-15)$ on the interval $[-8, 1]$. **SCORE: _____ / 7 PTS**

$$= x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$$

$$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - 10x^{-\frac{1}{3}} \quad \text{DNE @ } x = 0 \quad (1)$$

$$(1) \quad \frac{5}{3}x^{-\frac{1}{3}}(x-6) = 0 \quad @ \; x = 6 \notin [-8,1]$$

(1)

x	f(x)
(½) −8	−92
(½) 0	0
(½) 1	−14

ABSOLUTE MAX @ $x = 0$ (1)

MIN $x = -8$ (1)

+(½) FOR NO OTHER $x, f(x)$ VALUES

Evaluate the following limits. **SCORE: _____ / 10 PTS**

Your answer should be a number, ∞, –∞ or DNE (only if the first three answers do not apply).

[a] $\lim_{x \to 0^+} (1+x)^{2\cot x}$ 1^∞

$= \lim_{x \to 0^+} \left[e^{\ln(1+x)}\right]^{2\cot x}$

$= \boxed{e^{\lim_{x \to 0^+} \frac{2\ln(1+x)}{\tan x}}}$ 2

$= \boxed{e^2}$ $1\frac{1}{2}$

$\lim_{x \to 0^+} \frac{2\ln(1+x)}{\tan x}$ $\frac{0}{0}$

$= \boxed{\lim_{x \to 0^+} \frac{\frac{2}{1+x}}{\sec^2 x}}$ $1\frac{1}{2}$

$= \frac{\frac{2}{1+0}}{1^2}$

$= \boxed{2}$ 1

[b] $\lim_{x \to 0} \frac{e^{2x} - 2x - 1}{x(e^x - 1)}$ $\frac{0}{0}$

$1\frac{1}{2}$ $= \boxed{\lim_{x \to 0} \frac{2e^{2x} - 2}{e^x - 1 + xe^x}}$ $\frac{0}{0}$

$1\frac{1}{2}$ $= \boxed{\lim_{x \to 0} \frac{4e^{2x}}{e^x + e^x + xe^x}}$

$= \frac{4}{1+1}$

1 $= \boxed{2}$